Intermediate Report Nr. 12

Bernhard Geiger

August 20, 2009

Contents

1 The Interpolation Algorithm

As it was already shown in the first intermediate report, for values of $T_i$fulfilling the Nyquist theorem the envelope of the compound channel response can be estimated using the formula

$$|g(nT_i)|^2 \approx \frac{1}{T_i} z[n] \tag{1}$$

where $z[n]$ is the ED output, and $g(t)$ is the input to the ED (baseband channel, no modulation). Therefore it seems to be a good idea to use this approximation for a channel estimation, in which the estimate is the interpolation of discrete samples of the envelope. Such an estimate $|\hat{g}[n]|^2$, for example, is obtained by upsampling the output of the energy detector and perform interpolation filtering afterwards (as shown in Fig. 1). Upsampling is done by a factor of M, which determines the resolution T_r of the channel estimate according to

$$T_r = \frac{T_i}{M} \tag{2}$$

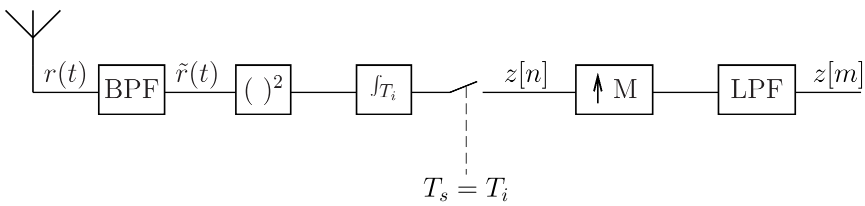


Figure 1: Interpolation Structures

Obviously, the approximation only holds for values of T_i impractical for most applications, but this fact only leads to the conclusion that the channel estimate will not be perfect, even in the noise-free case. Still, it is sensible to evaluate the performance of such an approach to find if acceptable performance can be obtained by disobeying the Nyquist theorem.

The quality of the channel estimate is evaluated using

$$Q_2 = 10 \log \frac{\sum_n \left(|\hat{g}[n]|^2 - |g[n]|^2\right)^2}{\sum_n \left(|g[n]|^2\right)^2} \tag{3}$$

where $|\hat{g}[n]|^2$ is the estimated high-rate PDP. As one can see, by setting the estimate to zero (i.e. setting $|\hat{g}[n]|^2 = 0\ \forall n$) the quality measure is equal to 0 dB. Any positive values of Q_2 therefore denote estimates which have less similarity with the actual channel than the zero estimate.

In the following, I will describe a few possibilities of obtaining the channel estimates and analyze their performance in a set of simulations.

1.1 sinc-Interpolation

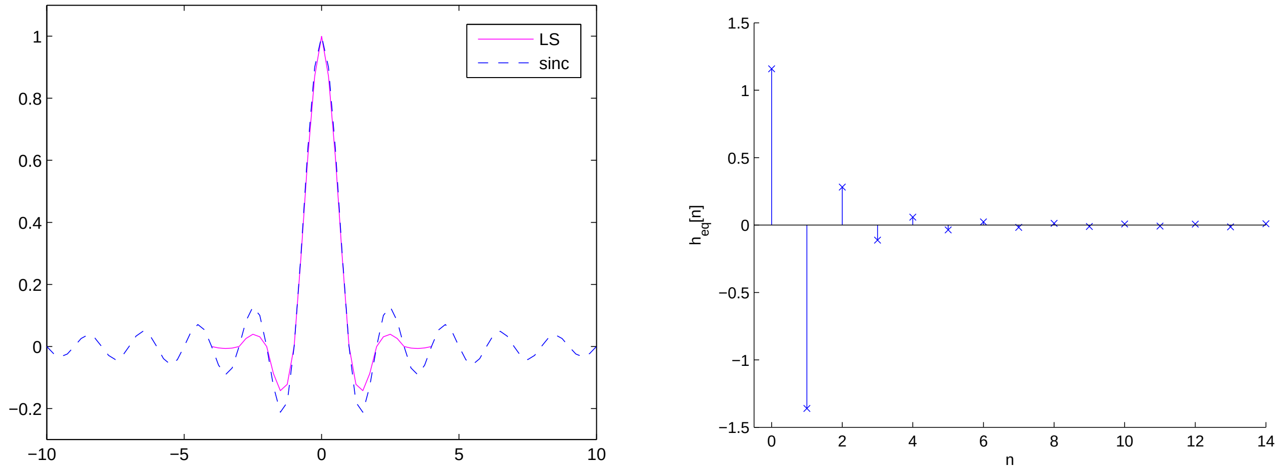
One straightforward way of interpolating the output samples of the ED is by filtering the upsampled signel with an ideal low-pass filter (which is equivalent to sinc interpolation). Unfortunately, this filter would require an infinite impulse response, which has to be truncated in real-world applications. Another possibility which was pursued in the simulations is to symmetrically pad the FFT of the signal with zeros[1]. If one is only interested in the magnitude of the signal one can omit zero-padding and only perform the $N_{reg}M$-point IFFT on the FFT of the ED output samples, where N_{reg} is the number of output samples after correlation and averaging.

Unfortunately, there is a problem with sinc-interpolation if the samples were not derived by Nyquist sampling (and generally, the ED output samples do not satisfy the Nyquist theorem). In these cases the interpolated signal can be seen as a Fourier series approximation or a low-pass filtered signal, which, after steep slopes, suffers from ringing. This effect is also well known from observing the low-pass approximation of a step function, where ringing causes the approximation to overshoot the top of the step function at first. Gibbs phenomenon describes this ringing to be independent of the cut-off frequency of the approximation.

Ringing, on the other hand, has the negative effect of separated local maxima prior to the leading edge, naturally more prominent for LOS cases (higher steps cause stronger ringing). These local maxima could lead to early leading edge detection, if the threshold is set to values low enough (e.g. due to inappropriate scenario identification)[2]. Moreover, ringing strongly influences the quality of the channel estimate.

[1] This is equivalent to upsampling and ideal low-pass filtering in the frequency domain – the time domain signal can be obtained with the inverse FFT.

[2] Although, for present simulations I could not find any indication that the influence of ringing is severe for practical ranging applications. Only for thresholds below $\zeta = 0.1$ LS interpolation outperforms sinc interpolation in terms of ranging accuracy.



(a) Comparison between sinc and LS interpolation filter

(b) Equalization filter

Figure 2: Filters used for interpolation algorithms

1.2 Least-squares Interpolation

The MATLAB Signal Processing Toolbox implements interpolation (`interp`) as a least squares (LS) operation, where the filter is adapted to the sequence to be interpolated (see Fig. 2(a)). One can assume that this approach has a much higher computational complexity, but one can also assume that the filter order (i.e. the support of the interpolation function) is much smaller. This also causes ringing to be somewhat limited, breeding the hope that ranging and channel estimation performance can be increased.

1.3 Equalization prior to Interpolation

Another idea which was proposed by Christian Vogel is to somehow equalize the influences of the integrator of the integrate and dump circuit. To do so, we have to look at the energy detector from a different point of view. If $\tilde{r}^2(t)$ denotes the input and $z[n]$ the output of the energy detector, we have

$$z[n] = \int_{nT_i - \frac{T_i}{2}}^{nT_i + \frac{T_i}{2}} \tilde{r}^2(t)dt \tag{4}$$

which is equivalent to

$$\begin{aligned} z[n] &= \int_{-\infty}^{nT_i + \frac{T_i}{2}} \tilde{r}^2(t)dt - \int_{-\infty}^{nT_i - \frac{T_i}{2}} \tilde{r}^2(t)dt & (5) \\ &= \int_{-\infty}^{nT_i + \frac{T_i}{2}} \tilde{r}^2(t)dt - \int_{-\infty}^{(n-1)nT_i + \frac{T_i}{2}} \tilde{r}^2(t)dt = \tilde{z}[n] - \tilde{z}[n-1] & (6) \end{aligned}$$

where $\tilde{z}[n] = \int_{-\infty}^{nT_i + \frac{T_i}{2}} \tilde{r}^2(t)dt$. This now is equivalent to a scenario in which the integrator is integrating continuously and after sampling the difference between consecutive samples is generated (see Fig. 3). If now $\tilde{R}(\jmath\omega)$, $\tilde{Z}(\jmath\omega)$ denote the continuous-time and $\tilde{Z}\left(e^{\jmath\theta}\right)$, $Z\left(e^{\jmath\theta}\right)$ the discrete-time Fourier transforms of the signals $\tilde{r}^2(t)$, $\tilde{z}(t)$, $\tilde{z}[n]$ and $z[n]$ we can describe the relationship between these signals

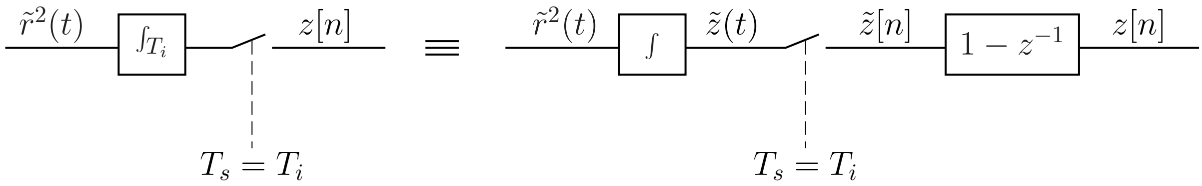


Figure 3: Energy Detector and equivalent structure

and/or spectra via

$$\tilde{Z}(\jmath\omega) = \frac{1}{\jmath\omega}\tilde{R}(\jmath\omega) \tag{7}$$

$$\tilde{z}[n] = \tilde{z}(nT_i) \tag{8}$$

$$Z\left(e^{\jmath\theta}\right) = \tilde{Z}\left(e^{\jmath\theta}\right) - \tilde{Z}\left(e^{\jmath\theta}\right)e^{-\jmath\theta} = \tilde{Z}\left(e^{\jmath\theta}\right)\left(1 - e^{-\jmath\theta}\right) \tag{9}$$

Given $z[n]$ and the assumption that the signal is sampled according to the Nyquist theorem – which is not fulfilled in practice[3] – one can calculate the samples of the input signal $\tilde{r}^2(t)$ of the energy detector by equalizing both the influences of the continuous-time integrator and the device computing the difference between consecutive samples. This way, one can obtain samples $\tilde{r}^2[n]$ of the input signal $\tilde{r}^2(t)$ and increase the accuracy of channel estimation. Plugging all equations together yields

$$\tilde{R}\left(e^{\jmath\theta}\right) = \frac{\jmath\theta}{T_i}\frac{1}{1 - e^{-\jmath\theta}}Z\left(e^{\jmath\theta}\right) \tag{10}$$

While the second term in this equation is a discrete accumulator, we have to design a filter to model the first term. This can be done using the formula for the inverse Fourier transform and the relationship $\int e^{cx}dx = \frac{e^{cx}}{c^2}(cx - 1)$ from [1]:

$$h_{eq}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\jmath\theta}{T_i}e^{\jmath\theta n}d\theta = \frac{\jmath}{2\pi T_i}\int_{-\pi}^{\pi}\theta e^{\jmath\theta n}d\theta = \frac{\jmath}{2\pi T_i}\left[\frac{e^{\jmath\theta n}}{\jmath^2 n^2}(\jmath\theta n - 1)\right]_{-pi}^{\pi} \tag{11}$$

$$= \frac{\jmath}{2\pi T_i}\left[\frac{e^{\jmath\pi n}}{\jmath^2 n^2}(\jmath\pi n - 1) - \frac{e^{-\jmath\pi n}}{\jmath^2 n^2}(-\jmath\pi n - 1)\right] \tag{12}$$

$$= \frac{\jmath}{2\pi T_i}\left[\frac{(-1)^n}{n^2}(1 - \jmath\pi n) - \frac{(-1)^n}{n^2}(\jmath\pi n + 1)\right] \tag{13}$$

$$= \frac{\jmath}{2\pi T_i}\frac{(-1)^n}{n^2}(1 - \jmath\pi n - \jmath\pi n - 1) = \frac{-2\jmath^2\pi n}{2\pi T_i}\frac{(-1)^n}{n^2} = \frac{(-1)^n}{nT_i} \tag{14}$$

This filter impulse response is then modeled by an FIR as shown in Fig. 2(b) and the samples to be interpolated are calculated according to

$$\tilde{r}^2[n] = \sum_{l=0}^{n}(z[l] * h_{eq}) \tag{15}$$

As it can be seen, this discrete accumulator is not stable because it has a pole at the unit circle. However, since the impulse response h_{eq} is zero-mean, the result of the convolution is also zero-mean *on average* because $z[n]$ is non-negative and thus the output of the discrete accumulator is bounded despite its instability. The overall equalizer structure thus can be seen in Fig. 4.

[3] It is only the question how much influence occurring aliasing has on the performance of the equalizer.

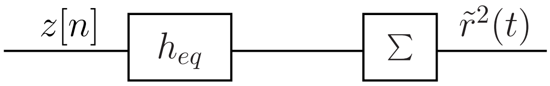


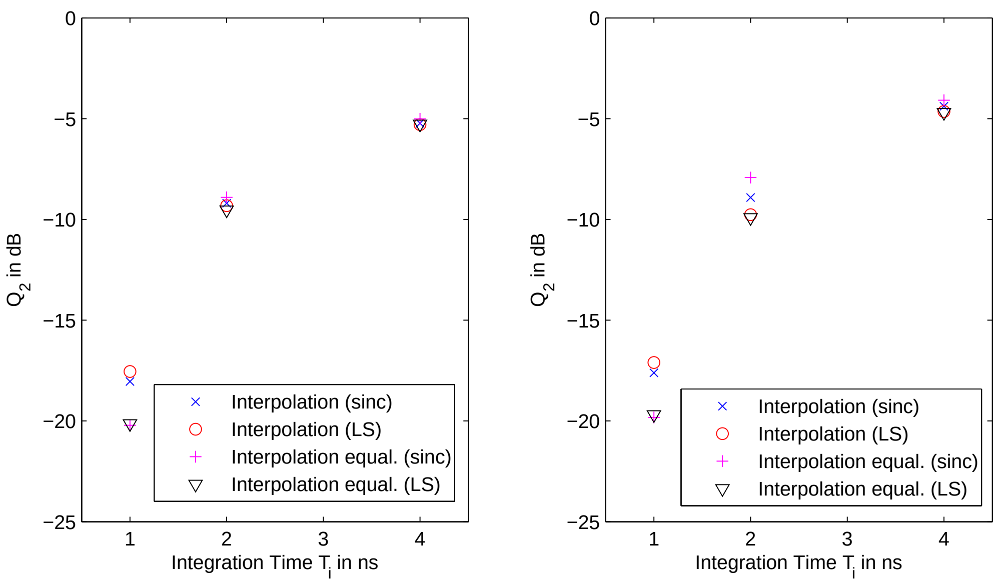
Figure 4: Equalizing integration effects

1.4 Results

To evaluate the performance of different approaches, I performed a set of simulations over 500 channels each for NLOS (k=0) and LOS (k=1) environments. The upsampling factor was set to achieve a temporal resolution of $T_r = 0.25$ ns. Both the quality of the channel estimate and the ranging accuracy were analyzed.

1.4.1 Channel Estimation

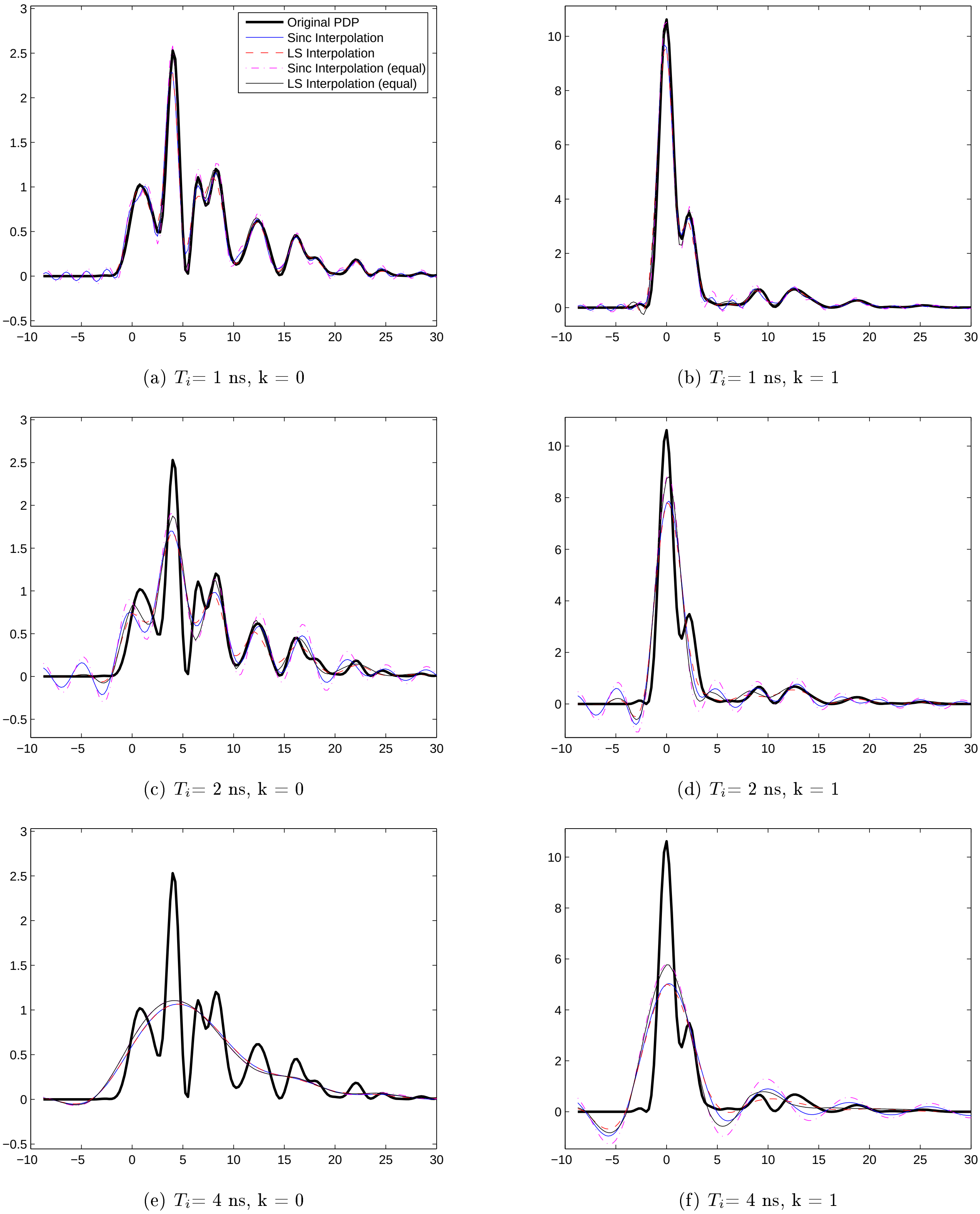
Fig. 5 shows the quality of the channel estimate for all possible integration periods for both LOS and NLOS scenarios. As expected, the quality of the channel estimate decreases strongly with increasing T_i, where only $T_i = 1$ ns manages to capture the dynamics of the channel to its full extent. Moreover, one can see that only this value for T_i benefits from prior equalization, since obviously larger integration periods introduce a significant amount of aliasing which cannot be corrected. Speaking of ringing, one can see that especially $T_i = 2$ ns benefits from limited support of the interpolation function (i.e. of finite order of the interpolation filter), because here LS interpolation outperforms sinc interpolation by far. For very large integration values of $T_i = 4$ ns the quality of the channel estimate is so bad that there is little difference to be observed between the possible interpolation techniques. These considerations are supported by Fig. 6 as well, which shows the channel estimates for the noise-free case for all possible values of T_i and k.



(a) k = 0 (b) k = 1

Figure 5: Quality of the channel estimate for the noise-free case

The influence of noise on the quality of the channel estimate can be seen in Fig. 7, where one can



(a) T_i= 1 ns, k = 0

(b) T_i= 1 ns, k = 1

(c) T_i= 2 ns, k = 0

(d) T_i= 2 ns, k = 1

(e) T_i= 4 ns, k = 0

(f) T_i= 4 ns, k = 1

Figure 6: Channel estimates for different values of T_i and k

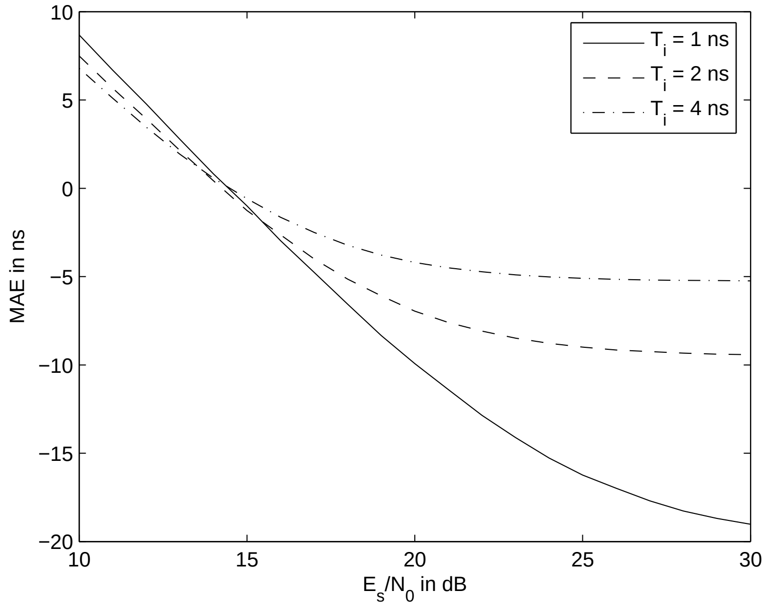


Figure 7: Quality of the channel estimate for different values of E_s/N_0 (equalized LS interpolation, k=0)

see that at least 15 dB of E_s/N_0 are required to outperform the zero estimate. Naturally, by increasing the SNR, the quality of the channel estimate improves as well, approaching the values depicted in Fig. 5 asymptotically.

1.4.2 Ranging

For ranging, there is little difference between LS and sinc interpolation, as Fig. 8 shows. Only the LOS case benefits from LS interpolation, which is also justified by increased channel estimation performance (*cf.* Fig. 5). Robustness, as it can be seen, is slightly worse for LS interpolation, and this effect is more pronounced for larger values of T_i.

Fig. 9 shows the ranging performance of the interpolation algorithms for the noise-free case. The range estimate was obtained using only MES-SB where the threshold ζ was varied between 0.03 and 0.9 only. It is once again shown that LS outperforms sinc interpolation for LOS scenarios (see Fig. 9(b)), where here surprisingly the equalized interpolation algorithms are in favor. For NLOS environments, performance is pretty similar for all algorithms, where again the equalized ones seem to perform a bit better (except for $T_i = 4$ ns). Performance decrease by increasing T_i is more pronounced for NLOS channels, whereas it can be overcome completely for LOS channels taking into account MES as a method of range estimation (for larger integration periods MES outperforms MES-SB, as it will be shown in IR 11).

References

[1] H.-J. Bartsch, *Mathematische Formeln.* Leipzig: VEB Fachbuchverlag, 1984.

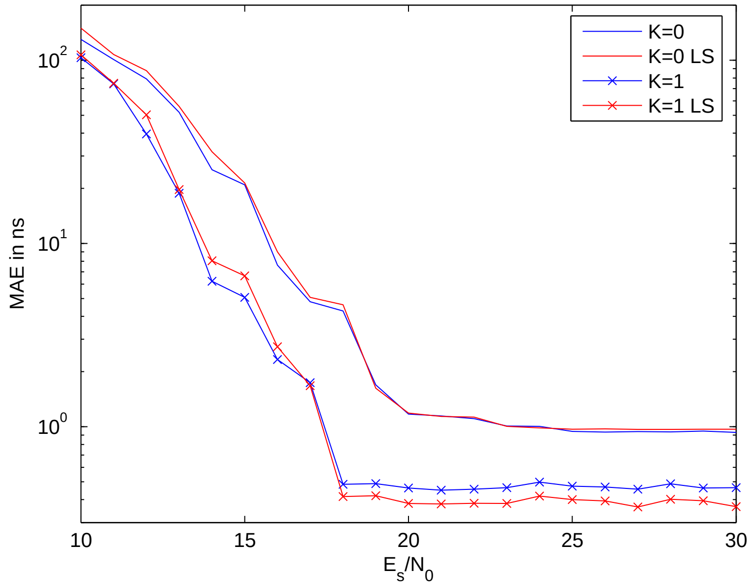
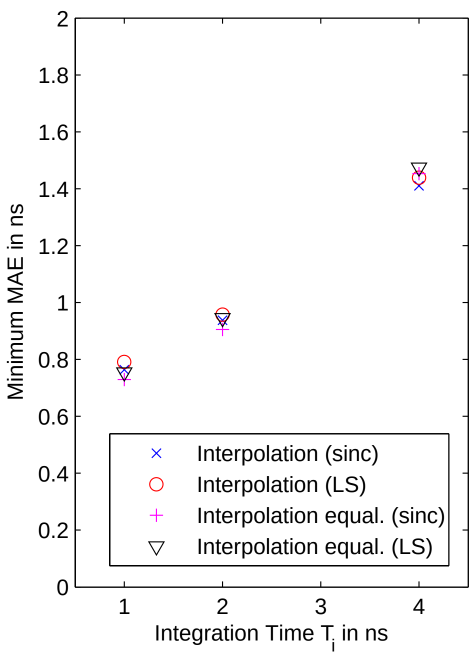
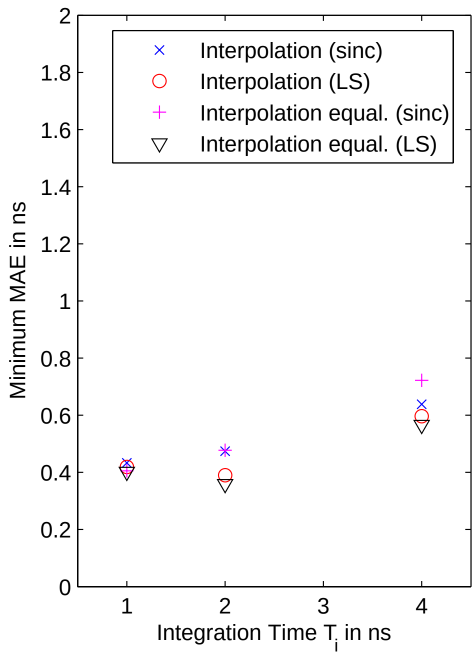


Figure 8: Ranging performance of the (unequalized) interpolation algorithm ($T_i = 2$ ns)



(a) k = 0



(b) k = 1

Figure 9: Ranging performance for the noise-free case