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1 Convex Optimization of the Sliding Algorithm

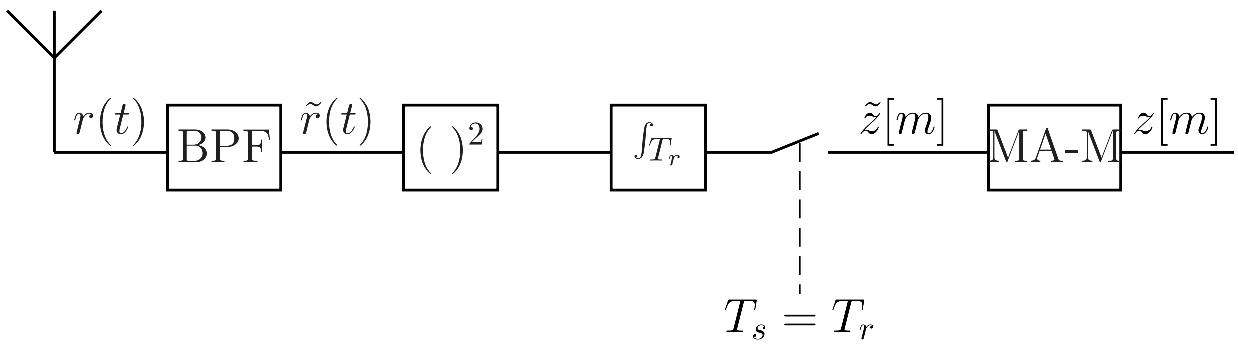


Figure 1: Direct Structure with MA filtering

As it was already shown in IR9, the sliding algorithm is equivalent to a high-rate direct ED structure followed by an MA filter of oder M, where

$$M = \frac{T_i}{T_r} \tag{1}$$

with $T_r = \text{GCD}\{T_C, T_i\}$. It was already shown in IR1 that this high resolution satisfies the approximation developed in [1], making this approach suitable for channel estimation. As a consequence, according to Fig. 1, the relationship between $\tilde{z}[m]$ and $z[m]$ can be given with

$$z[m] = h_{MA} * \tilde{z}[m] \tag{2}$$

or, equivalently

$$\mathbf{z} = \mathbf{H}_{MA}\tilde{\mathbf{z}} \tag{3}$$

where $\mathbf{H}_{MA}$ is the convolution matrix of the MA filter. Unfortunately, this matrix is singular (the corresponding z-transform has zeros on the unit circle), so a direct equalization of the filter operation is not possible. As a consequence, equalization can only be done in a iterative sense by minimizing some cost function. This cost function may be of different design, leading to a maximum likelihood, least squares, or any other solution of the problem. As it is widely known, the ML solution is identical to the LS solution for the noise-free case and the case where the noise is white and Gaussian. However, the LS solution leads to the well known MMSE equalizer, which requires some artificial design noise variance as stated in IR9.

Another solution to the upper problem is a solution in the ML sense. Such a solution can be obtained using the Expectation-Maximization (EM) algorithm introduced by [?]. Unfortunately, is not guaranteed to converge to a global optimum and therefore may stall at some local optima. A series of experiments in which the EM algorithm was used for curve fitting (fitting Gaussian kernels to a curve describing the channel response) failed for cases where MPCs are closely spaced. It also turned out that the initial position of the approximation kernels strongly influences the convergence and optimality of the solution.

As a final try, the author resigned to using predefined toolboxes in MATLAB, namely the `cvx` toolbox developed by Stephen Boyd and TODO. This toolbox implements iterative convex optimization of constrained problems, which allowed me to define my problem as follows:

$$\hat{\tilde{\mathbf{z}}} = \arg\min_{\tilde{z}} \{\max\{\mathbf{z} - \mathbf{H}_{MA}\tilde{\mathbf{z}}\}\} \quad (4)$$

$$\text{subject to:} \quad (5)$$

$$\hat{\tilde{\mathbf{z}}} \geq 0 \quad (6)$$

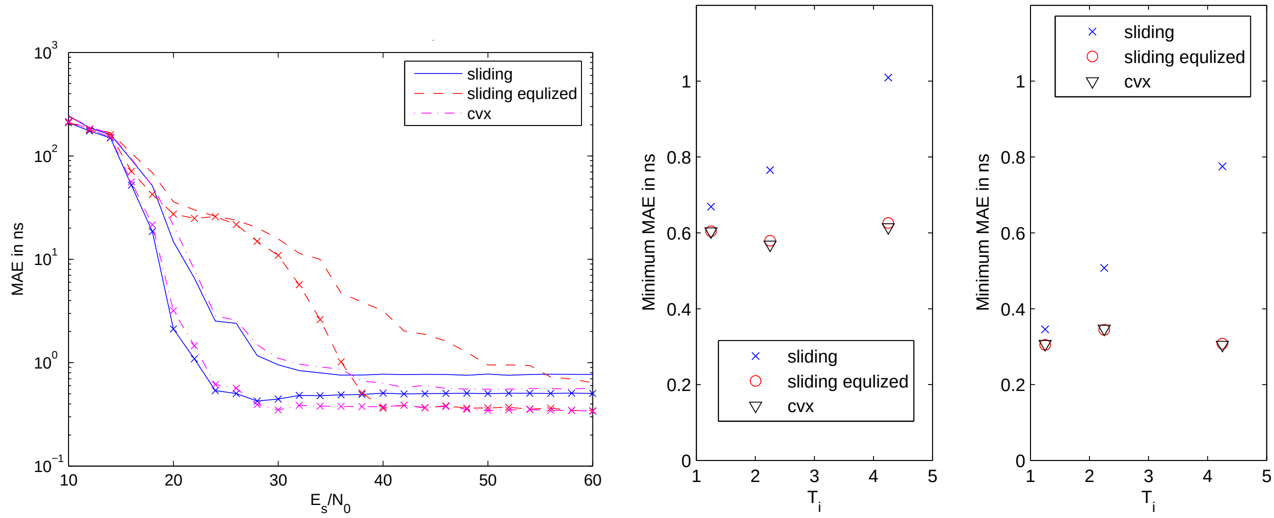
This problem, also known as Chebyshev minimization or Chebyshev minimax problem minimizes the ℓ_∞ norm of the error signal (compared to LS, which minimizes the ℓ_2 norm). Strictly speaking, optimization is done not on the outputs of the ED $z[m]$ but on the PDP estimate $p[m]$, which makes the non-negativity constraint invalid. However, as experiments showed, this constraint is necessary to guarantee stability.

1.1 Simulations and Results

I performed a set of simulations over each 250 channels for LOS and NLOS environments. Integration periods T_i were chosen from 1.25, 2.25 and 4.25 ns, achieving a resolution of $T_r = 0.25$ ns. To reduce computational effort, convex optimization and equalization according to IR9 was limited to 40 ns preceding and 80 ns succeeding the maximum energy block (MES) of the sliding algorithm output. As it can be seen in Figs. 2 and 3, this limitation somewhat improves robustness of the equalized sliding algorithm, despite this gain in robustness still does not yield acceptable ranging accuracy for a wide range of SNR. Interestingly, ranging accuracy of the constrained Chebyshev minimization (CVX) on one hand has a robustness comparable to the unequalized sliding algorithm, whereas it has an accuracy comparable to the equalized sliding algorithm. This makes the CVX sliding algorithm an optimal choice, if the computational complexity can somehow be reduced. In terms of channel estimation similar considerations hold: whereas equalized sliding outperforms sliding for high values of E_s/N_0, CVX sliding manages to obtain best results for the whole SNR region under consideration (see Fig. 3(a)).

References

[1] H. Urkowitz, “Energy detecction of unknown deterministic signals,” *Proceedings of the IEEE*, vol. 55, no. 4, pp. 523–531, April 1967.

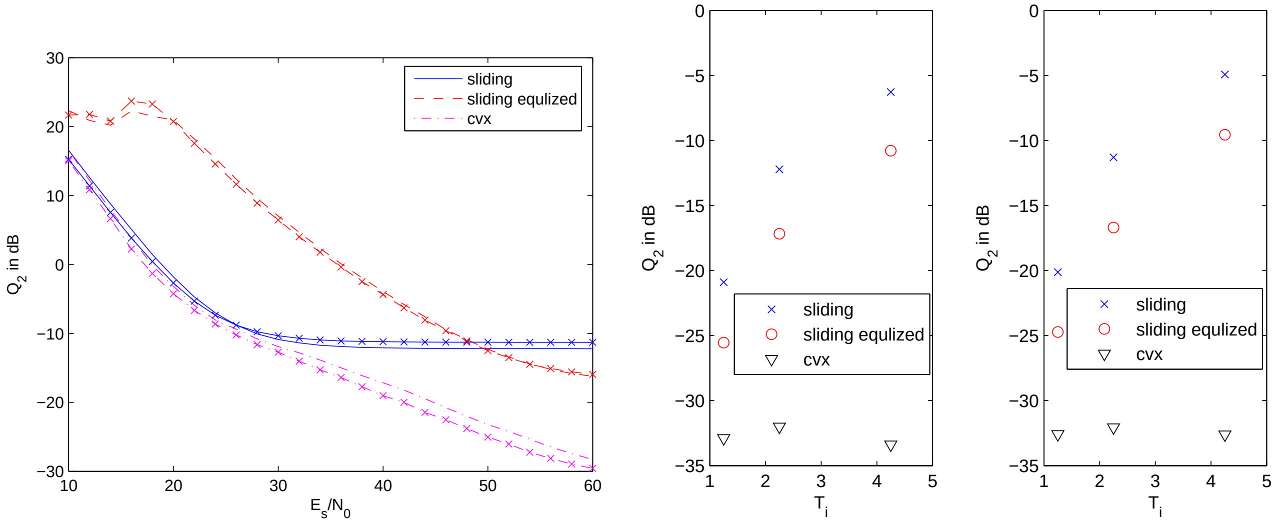


(a) Ranging Performance for $T_i = 2$ ns, k=0

(b) Ranging Performance for k=0 (noise-free)

(c) Ranging Performance for k=1 (noise-free)

Figure 2: Ranging Performance for Sliding Algorithms



(a) Channel Estimation for $T_i = 2$ ns, k=0

(b) Channel Estimation for k=0 (noise-free)

(c) Channel Estimation for k=1 (noise-free)

Figure 3: Accuracy of the channel estimate for Sliding Algorithms