Intermediate Report Nr. 2

Bernhard Geiger

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1 How to chose T_s and T_i?

Starting from IR1 we found that

$$|\hat{g}(t)| = \sum_i |g(iT_s)| \text{sinc}\left(\frac{t - iT_s}{T_s}\right) \tag{1}$$

is a good approximation for the envelope of the compound CIR if

- $T_s < \frac{1}{2W}$, thus fulfills the Nyquist criterion and
- $T_i = \frac{1}{2W}$, thus Urkowitz's approximation holds.

The first problem with this optimal approach is that it would require two parallel integrators, since the integration period is larger than the sampling period. In order to reduce receiver complexity, I would like to find out if we can relax these requirements a bit in order to reduce required sampling rates and processing power. Naturally, this can only be done with the cost of reduced performance. In the following, I will discuss three possibilities to relax the requirements: increasing T_s, increasing T_i and increasing both of them. Performance is tested with respect to the following criteria: mean-square estimation error Q_2, mean absolute error in detecting the maximum energy peak (MAE-MES) and mean absolute error in detecting the leading edge (MAE-LE, threshold ζ was set to 20% of the maximum peak). MES and LE detection where performed on the reconstructed envelope of the compound CIR. A set of 1000 channel realizations were simulated with a Ricean k-factor of 0.2 and an RMS delay spread of 10 ns. Simulation results are illustrated in Tab. 1

1.1 Sub-Nyquist Sampling

In this case we use a reduced sampling rate, or an increased sampling period. As a first step, we could slightly increase the sampling interval so that integration intervals do not overlap anymore. By setting $T_i = T_s = 1.25$ ns either only one integrator is required or two integrators can sample at an even more relaxed rate (every 2.5 ns instead of every 2 ns). The price for this, however, is an increased approximation error by about 2 dB. By even further increasing the sampling period to $T_s = 2$ ns a bigger error is introduced to the system, now clearly caused by not satisfying the Nyquist theoreme (see Fig.2). Subsequently, performance is decreased by almost one order of magnitude (9 dB) with respect to approximation gain.

Even more important in terms of ranging is the performance of selecting the maximum energy peak or the leading edge. Even the optimal setting lead to an error of approximately 20 ns for each detection, which is slightly below the minimum achievable $\frac{T_s}{4}$ for operating directly on ED outputs. By increasing the sampling period to 1.25 ns MES detection yields much worse results, whereas LE detection still performs acceptable.

1.2 Increased Integration Periods

Increasing integration periods itself has little capabilities of reducing the receiver's complexity, but maybe it will turn out to improve performance in terms of better noise averaging (which hopefully will be analyzed later). At this point, increasing the integration periods is necessary to prepare the path to the next step, where both integration period and sampling rate will be relaxed in order to obtain a low-complexity receiver. Thus, in this subsection I would like to analyze the effects of increased integration periods on the validity of Urkowitz' approximation, especially if the integration period is a non-integer multiple or a fraction of the ideal integration period.

Starting from

$$y_i = \int_{iT_s-\frac{T_i}{2}}^{iT_s+\frac{T_i}{2}} g^2(t)dt \approx \frac{1}{2W}\sum_{k=0}^{2WT_i} g_k^2, \tag{2}$$

where g_k are equidistant samples from the interval T_i, we can simplify our analysis by setting

$$\int_{iT_s-\frac{T_i}{2}}^{iT_s+\frac{T_i}{2}} g^2(t)dt = T_i \mathrm{Avg}\left\{g^2(t)\right\} \; \forall t \in \left[iT_s - \frac{T_i}{2}, iT_s + \frac{T_i}{2}\right]. \tag{3}$$

That is, we approximate the signal within the integration boundaries with its mean value. This assumption holds better if the integration interval is short compared to the time during which the signal undergoes no or little changes. That, on the other hand implies that all the samples g_k are identical within the interval T_i:

$$y_i = T_i \mathrm{Avg}\left\{g^2(t)\right\} \approx \frac{1}{2W} 2WT_i g^2 \tag{4}$$

If we now say that $T_{i,opt} = \frac{1}{2W}$, we can state:

$$y_i = T_i \mathrm{Avg}\left\{g^2(t)\right\} \approx T_i g^2 \tag{5}$$

$$\frac{y_i}{T_i} = \mathrm{Avg}\left\{g^2(t)\right\} \approx g^2 \tag{6}$$

This last equation makes it very clear what Urkowitz' approximation means if we are using only one sample as an estimate. Furthermore, it clearly shows that this approximation holds better, the smaller T_i is! In the limiting case where $T_i \to 0$ we are facing the fact that we are indeed sampling the compound response at certain values. If then our sampling period is short enough (i.e. if Nyquist is satisfied), we are getting optimum results.

This means that the output of the energy detector has to be scaled with the actual integration period in order to make the approximation valid. Naturally, the approximation yield worse and worse results, the longer the integration time is. Reducing the integration period has some beneficial effect on the performance of the approximation, since shorter a shorter T_i does less averaging than a long T_i.

Urkowitz' approximation holds the better the higher the number $2WT_i$ is. Still, I would like to pick one particular case from Tab. 1 and give an explanation why the previous statement is not helpful in our case: Setting the integration period to 2.5 ns, the product $2WT_i$ gets exactly 2. This means that the output of the energy detector now approximates the sum of 2 samples from that interval. The problem now is that we have no means to find out *how* these two samples contribute to the ED output, that is we do not know which of these samples contained more energy. In the simulations I assumed both of these samples to be identical. The weight for the according sinc-function then was the mean value of these two samples.

Looking at the Fig. 1 one can see that for a sufficient sampling period of 1 ns the error increases with increasing integration intervals. Especially if the integration period is set to 2.5 ns, where the ED output represents two samples from within that interval (but which cannot be resolved), the approximation error is large. Similar can be said about detection errors.

1.3 Relaxing Sampling and Integration Requirements

Up to this point we only either increased the integration or the sampling period. In both cases an increase led to the result that the ED output could not fully reflect dynamics in the original signal, either because the sampling theoreme was not fulfilled or because the samples represented by the ED output could not be reconstructed (because one output was mapped to two samples). This section now is devoted to the performance analysis of systems which have relaxed requirements for both timing parameters.

As an example one may propose that if we already increased the sampling period and thus disobeyed the Nyquist theoreme, that we could also use longer integration periods, so that at least each sinc-function is weighted by accumulated information from the whole interval it will finally represent. And indeed, for a sampling period of 2 ns performance could be increased by increasing the integration interval to 2 ns as well. Increasing the integration period even more on the other hand introduces more errors. Obviously, the best results can be obtained by setting the integration interval identical to the sampling period – in this case, the according sinc-function is weighted exactly with the information from within its actual sampling interval.

This only holds if the sampling interval is already greater than the Nyquist theoreme would suggest. If we satisfy the Nyquist theoreme (e.g. a sampling period of 1 ns) the error reduces with shorter integration periods – in such cases shorter integration periods resemble more and more an actual sampling process, which is desirable if the sampling period is short enough. That is, as long as a certain number of periods from the carrier frequency fall within the integration interval. This also explains the curves in Fig. 3, especially why performance decreases if $T_i < 0.3$ ns. Alas, to this point I haven't found out yet why the optimum for $T_s = T_i = 2$ ns is so pronounced, and why for $T_s = T_i = 1$ ns detection of MES and LE is slightly worse.

However, it is pretty clear why for MES lower values of T_i perform worse: The smaller the integration period is compared to the sampling period, the higher is the probability that a different sample qualifies as MES, even if it's not. LE detection on the other hand in this case does not rely on MES detection beforehand, but just uses threshold comparison. It therefore needs only a highly accurate approximation of the leading edge[1].

[1]These are things which *might not* be of interest for my diploma thesis - however, I just wanted to mention them briefly, although I did not understand it to its full extent yet.

T_s ns	T_i ns	Q_2 dB	MAE-MES ns	MAE-LE ns
1.00	1.00	-22.05	0.20	0.17
1.00	1.25	-20.35	0.20	0.17
1.00	2.50	-12.95	0.60	0.61
1.25	1.25	-18.48	0.38	0.20
1.25	2.50	-13.06	0.53	0.59
2.00	1.25	-11.61	1.32	0.66
2.00	2.00	-12.42	0.83	0.48
2.00	2.50	-11.80	0.81	0.74
2.50	2.50	-10.26	1.09	1.09
1.00	1.00	-22.35	0.13	0.13
1.00	1.25	-20.33	0.15	0.15
1.00	2.50	-13.19	0.69	0.49
1.25	1.25	-18.50	0.44	0.20
1.25	2.50	-13.30	0.62	0.50
2.00	1.25	-12.11	0.86	0.39
2.00	2.00	-12.43	0.86	0.46
2.00	2.50	-12.20	0.78	0.60
2.50	2.50	-10.73	1.10	0.71
1.00	1.00	-21.17	0.06	0.22
1.00	1.25	-19.76	0.03	0.20
1.00	2.50	-12.00	0.18	0.68
1.25	1.25	-18.30	0.13	0.24
1.25	2.50	-12.06	0.11	0.66
2.00	1.25	-10.79	0.56	0.81
2.00	2.00	-12.17	0.15	0.54
2.00	2.50	-10.97	0.10	0.84
2.50	2.50	-8.99	0.37	1.32

Table 1: Performance of different T_s and T_i for Ricean k factors of 0.2, 0 and 0.8

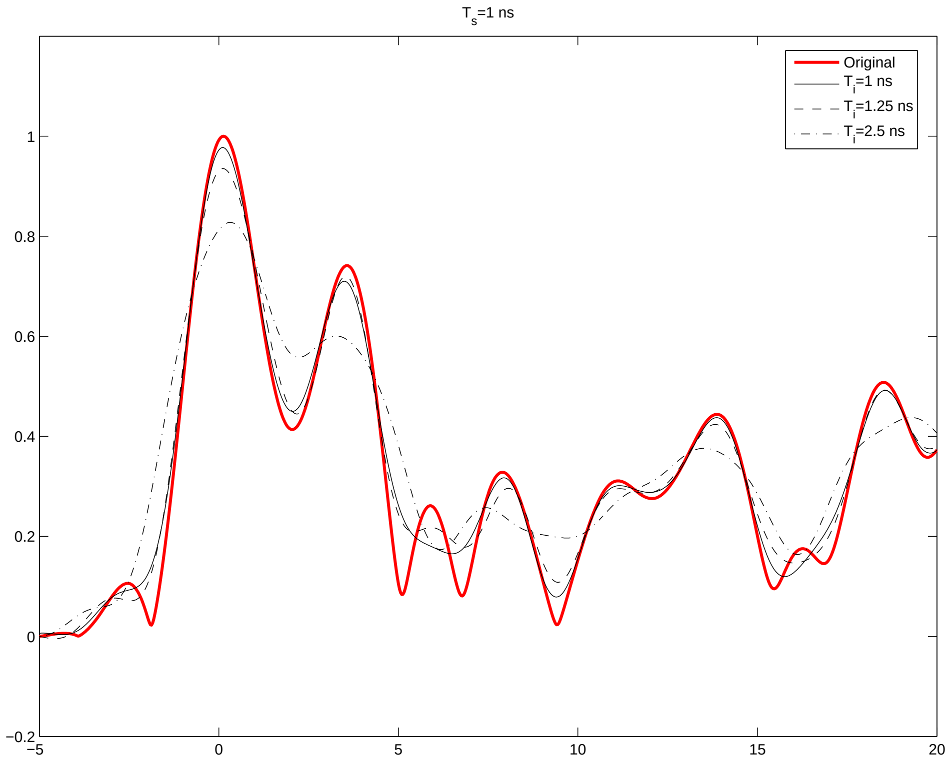

Figure 1: Approximation with $T_s = 1$ ns

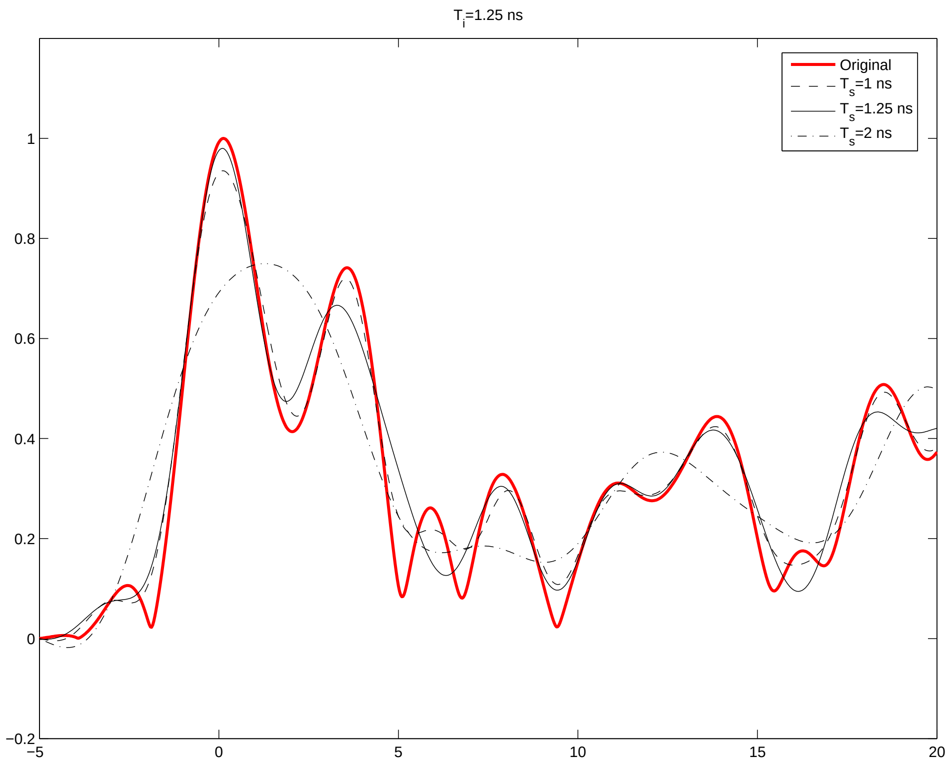

Figure 2: Approximation with $T_i = 1.25$ ns

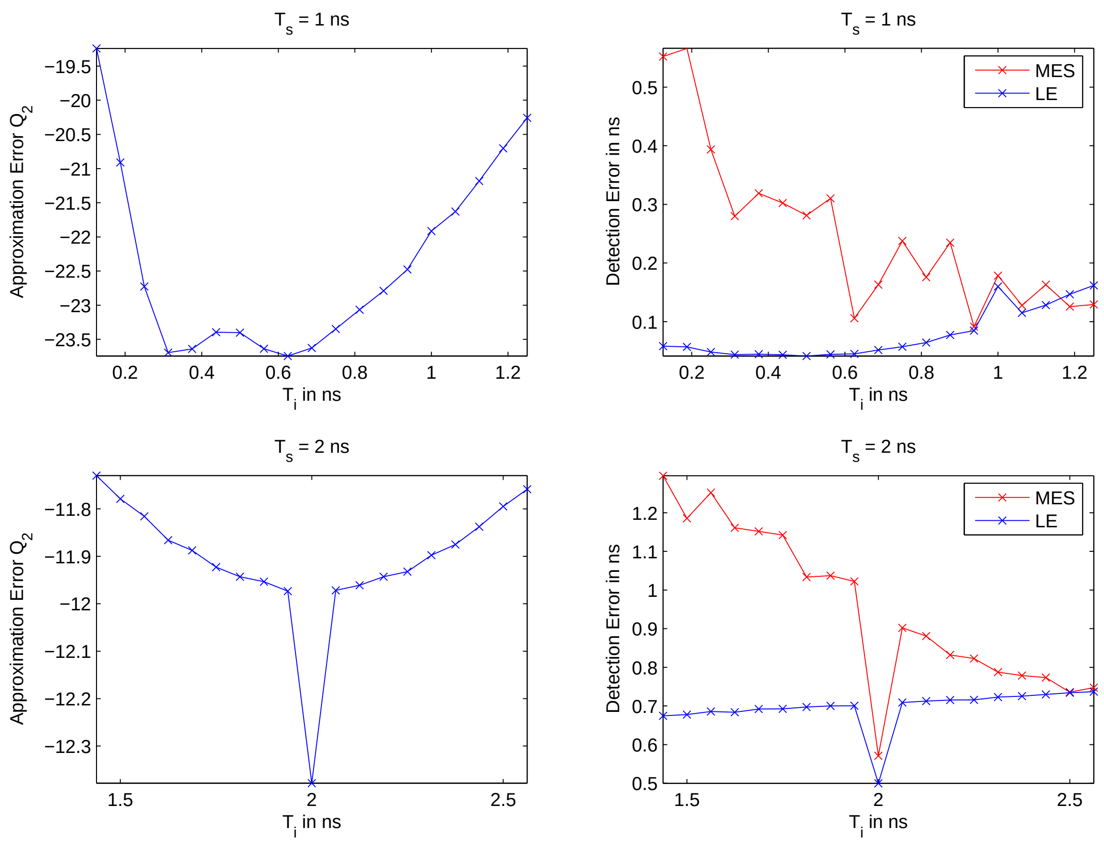

Figure 3: Optimal T_i for given T_s