Intermediate Report Nr. 8

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Contents

1 The Parallel Ranging Algorithm

Using the parallel structrure depicted in Fig. 1 one can use two EDs and correlators simultaneously at half the sampling rate, thus reducing the computational complexity of the overall system. In order to obtain the same temporal resolution as for the direct ranging scheme, the input signal to the parallel structure has to be delayed by half the integration period. That is, if an integration period of T_i is chosen, the input to the second branch of the system has to be delayed by $\frac{T_i}{2}$ – and that's exactly the resulting temporal resolution T_r. For a fair comparison, thus, a parallel structure with T_i has to be compared to a direct structure employing $\frac{T_i}{2}$.

Furthermore, there are at least two possibilities to work on such a parallel structure: One could perform leading edge detection on either of the two paths independently (that is, operating with $p_1[n]$ and $p_2[n]$) and then combine the estimates. Or, the paths could be combined first (obtaining $p[m]$ at a sampling rate of $\frac{T_i}{2}$) and leading edge detection would be performed on the outcome of this combination. For the SB algorithms, these two solutions are identical, if the combination of the estimates is limited to choosing the earlier time value (which seems most sensible):

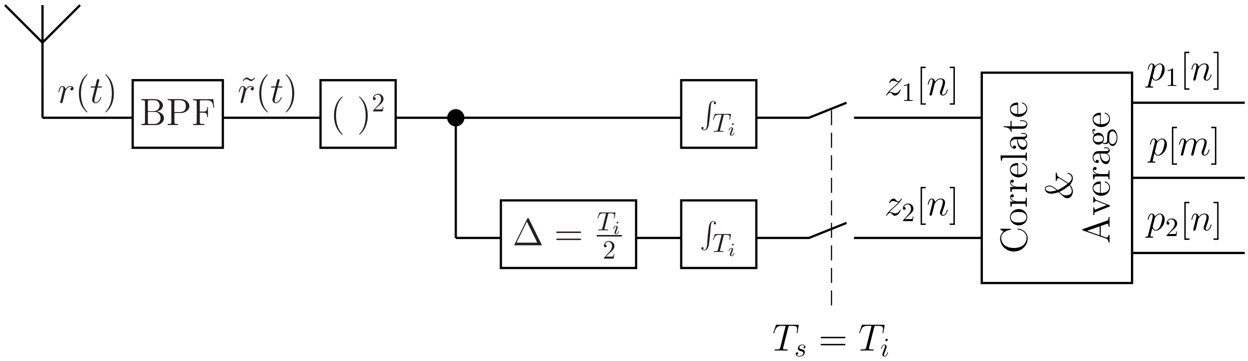

Figure 1: Parallel ED Structure

Assuming that $p_1[n] = p[2m]$ and $p_2[n] = p[2m+1]$, we can say

$$\hat{m}_{SB} = \min \{ m \in (m_{max} - w_{SB}, m_{max}) \, | p[m] > \zeta \}, \quad (1)$$

where m_{max} is the index of the maximum of $p[m]$ and w_{SB} is the search-back window size in samples of m. This result is identical to

$$\hat{n_1}_{SB} = \min \{ n \in (n_{1_{max}} - \tilde{w}_{SB}, n_{1_{max}}) \, | p_1[n] > \zeta \} \quad (2)$$

$$\hat{n_2}_{SB} = \min \{ n \in (n_{2_{max}} - \tilde{w}_{SB}, n_{2_{max}}) \, | p_2[n] > \zeta \} \quad (3)$$

$$\hat{m}_{SB} = 2 \min \left\{ \frac{\hat{n_1}_{SB}}{2}, \frac{\hat{n_2}_{SB} - 1}{2} \right\}, \quad (4)$$

where $n_{i_{max}}$ is the index of the maximum of $p_i[n]$ and $\tilde{w}_{SB}$ is the search-back window size in samples of n.

For MES, there is a slight difference, however: If we only obtain the estimates from the two paths, all we can do is to build an average. If the paths are combined prior to ME selection, one can either choose the actual MES or build a weighted average between the MES's of both paths (that's how I implemented it).

Thomas Gigl suggested that it would be a possibility to perform some sinc-interpolation between the ED samples – however, it is not absolutely clear how to do so, since there are again two possibilities: Combine first, interpolate later; or: interpolate, estimate and combine the estimates. sinc-interpolation is a thing I would like to try in the near future, whereas I selected linear interpolation as a first improvement of the parallel structure.

1.1 Proof, that parallel structure is identical to the direct structure with a MA filter

Starting from the parallel algorithm where we obtain the ED outputs $z_1[n]$ and $z_2[n]$ we can derive the following expressions for its values[1]. As we know, each ED path integrates and samples with a period of T_i, whereas the lower path operates on a received signal shifted by $\frac{T_i}{2}$. Therefore:

$$z_1[n] = \int_{nT_i-\frac{T_i}{2}}^{nT_i+\frac{T_i}{2}} \tilde{r}^2(t)dt \tag{5}$$

$$z_2[n] = \int_{nT_i-\frac{T_i}{2}}^{nT_i+\frac{T_i}{2}} \tilde{r}^2\left(t-\frac{T_i}{2}\right)dt = \int_{nT_i-\frac{T_i}{2}+\frac{T_i}{2}}^{nT_i+\frac{T_i}{2}+\frac{T_i}{2}} \tilde{r}^2(t)dt \tag{6}$$

If we now assume that these T_i-rate outputs of both detectors are combined to a single $\frac{T_i}{2}$-rate output $z[m]$, we can state without loss of generality that the upper path generates the even, whereas the lower path generates the odd samples of the combination.

$$z_1[n] = z[2m] \tag{7}$$

$$z_2[n] = z[2m+1] \tag{8}$$

Having further that due to the changed rate $n = 2m$, we can continue

$$z[2m] = \int_{2m\frac{T_i}{2}-\frac{T_i}{2}}^{2m\frac{T_i}{2}+\frac{T_i}{2}} \tilde{r}^2(t)dt \tag{9}$$

$$z[2m+1] = \int_{2m\frac{T_i}{2}-\frac{T_i}{2}+\frac{T_i}{2}}^{2m\frac{T_i}{2}+\frac{T_i}{2}+\frac{T_i}{2}} \tilde{r}^2(t)dt = \int_{(2m+1)\frac{T_i}{2}-\frac{T_i}{2}}^{(2m+1)\frac{T_i}{2}+\frac{T_i}{2}} \tilde{r}^2(t)dt \tag{10}$$

$$z[m] = \int_{m\frac{T_i}{2}-\frac{T_i}{2}}^{m\frac{T_i}{2}+\frac{T_i}{2}} \tilde{r}^2(t)dt \tag{11}$$

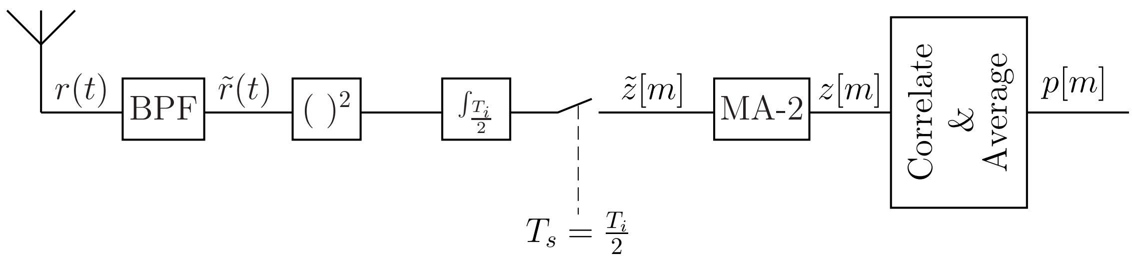

Figure 2: Direct structure followed by MA filter

The direct structure as shown in Fig. 2 on the other hand, which is operates at an integration and sampling period of $\frac{T_i}{2}$ provides the ED output samples $\tilde{z}[m]$ according to the formula

$$\tilde{z}[m] = \int_{m\frac{T_i}{2}-\frac{T_i}{4}}^{m\frac{T_i}{2}+\frac{T_i}{4}} \tilde{r}^2\left(t-\Delta\right)dt, \tag{12}$$

[1] In fact, since correlation and averaging are linear operations, the following derivations also hold if they would be performed on the PDP estimates $p_1[n]$ and $p_2[n]$, respectively. However, since we want to show the identity regardless of consecutive operations, the derivation is done with respect to the ED outputs.

where Δ is an arbitrary shift of the initial integration instant relative to the parallel structure. Having a two-tap MA filter with an impulse response of $h[m] = \delta[m] + \delta[m-1]$, we get the filter output according to

$$\begin{aligned} z[m] &= \tilde{z}[m] + \tilde{z}[m-1] && (13)\\ &= \int_{m\frac{T_i}{2}-\frac{T_i}{4}}^{m\frac{T_i}{2}+\frac{T_i}{4}} \tilde{r}^2\,(t-\Delta)\,dt + \int_{(m-1)\frac{T_i}{2}-\frac{T_i}{4}}^{(m-1)\frac{T_i}{2}+\frac{T_i}{4}} \tilde{r}^2\,(t-\Delta)\,dt && (14)\\ &= \int_{m\frac{T_i}{2}-\frac{T_i}{4}}^{m\frac{T_i}{2}+\frac{T_i}{4}} \tilde{r}^2\,(t-\Delta)\,dt + \int_{m\frac{T_i}{2}-\frac{T_i}{4}-\frac{T_i}{2}}^{m\frac{T_i}{2}+\frac{T_i}{4}-\frac{T_i}{2}} \tilde{r}^2\,(t-\Delta)\,dt && (15)\\ &= \int_{m\frac{T_i}{2}-\frac{T_i}{4}}^{m\frac{T_i}{2}+\frac{T_i}{4}} \tilde{r}^2\,(t-\Delta)\,dt + \int_{m\frac{T_i}{2}-3\frac{T_i}{4}}^{m\frac{T_i}{2}-\frac{T_i}{4}} \tilde{r}^2\,(t-\Delta)\,dt && (16) \end{aligned}$$

Now the lower integration bound of the first integral is identical to the upper bound of the second one, so we can combine these integrals to one:

$$z[m] = \int_{m\frac{T_i}{2}-3\frac{T_i}{4}}^{m\frac{T_i}{2}+\frac{T_i}{4}} \tilde{r}^2\,(t-\Delta)\,dt = \int_{m\frac{T_i}{2}-\frac{T_i}{2}}^{m\frac{T_i}{2}+\frac{T_i}{2}} \tilde{r}^2\left(t-\frac{T_i}{4}\right)dt \qquad (17)$$

Accordingly, the output of the parallel structure is identical to the output of the direct structure followed by a two-tap MA filter, if the input to the direct structure is shifted by $\Delta = \frac{T_i}{4}$ (see Fig. 3). This shifting can be corrected, so there indeed is an identity which can be exploited to increase ranging accuracy.

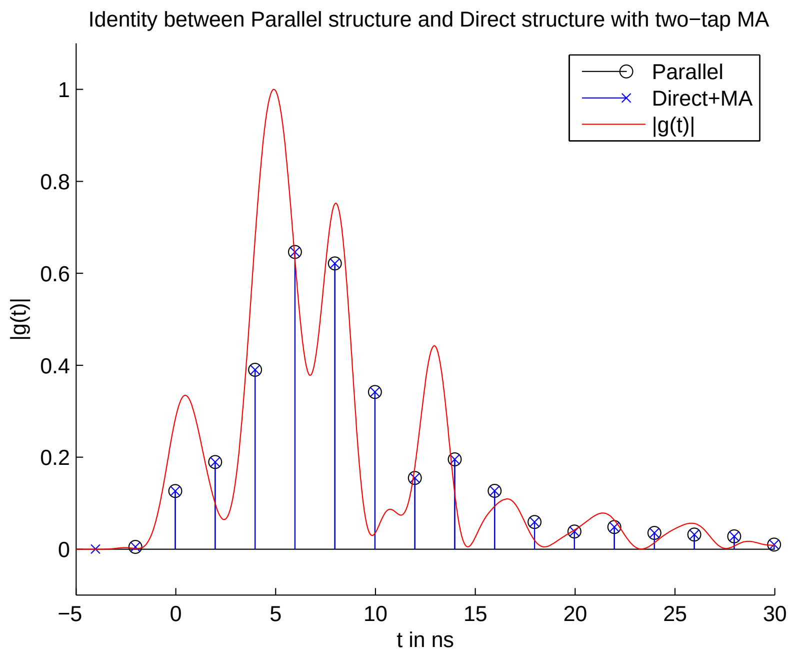

Figure 3: Proof of identity

1.2 Linear Interpolation

For linear interpolation, the outputs of the EDs have to be combined prior to estimation, since not only the time instants but also the actual values have to be available. These values can then be used together with the time instants to perform a linear interpolation (fit a line through two points as seen in Fig. 4) and for refining the ranging result. Then, the refined esimate is calculated as the intersection between the fitted line and the thresholding line. The two points I chose for linear interpolation are the PDP estimations $p[\hat{m}_{SB}]$ and $p[\hat{m}_{SB}-1]$. With $\tau = \hat{m}_{SB}T_r$ we can then say that

$$p(\tau) = k\tau + d \tag{18}$$

$$p(\tau - T_r) = k(\tau - T_r) + d. \tag{19}$$

That is, we fit a line through the first threshold exceeding energy block and its immediate predecessor. Reordering these terms we can state that

$$k = \frac{p(\tau) - p(\tau - T_r)}{\tau - \tau + T_r} = \frac{p(\tau) - p(\tau - T_r)}{T_r} \tag{20}$$

$$d = p(\tau) - k\tau \tag{21}$$

Intersecting this line with the threshold, we obtain the refined TOA estimate $\hat{\tau}$:

$$\hat{\tau} = \frac{\zeta - d}{k} \tag{22}$$

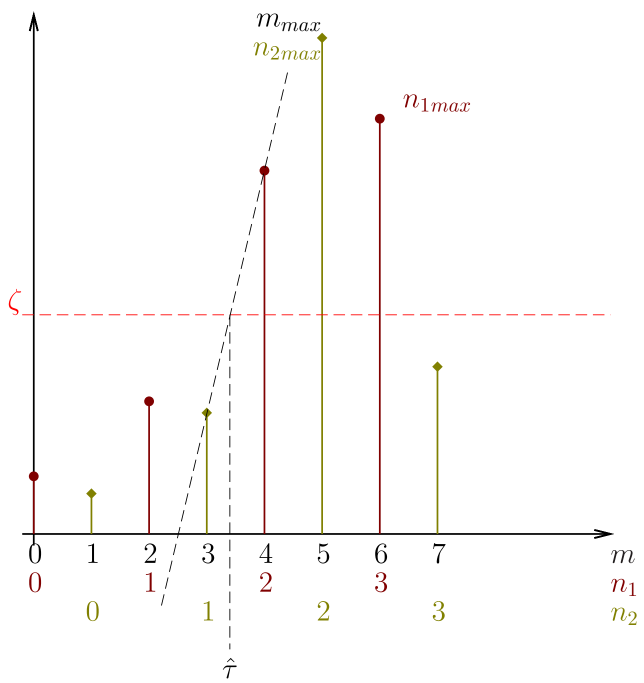

Figure 4: Linear Interpolation

To evaluate the performance of linear interpolation I ran a set of simulations for both LOS and NLOS scenarios ($k = 0$ and $k = 1$ 250 simulations each, and 100 simulations for both CM1 and CM2). I further chose a search-back window size of 32 ns and integration periods of 4 and 8 ns. The results are shown in Fig. 5, where for each and every SNR value the optimum threshold was chosen.

As it can be seen, linear interpolation does not yield any improvement of accuracy – in fact, in most cases the accuracy worsens significantly. This can be explained by the fact, that the energy block first exceeding the threshold will most likely contain the leading edge as a whole, so any search in front of this block will lead to a wrongly calculated TOA. Furthermore, due to linearization the optimum threshold rises to much greater values, since in order to obtain the correct result, the first block choice should be the block immediately after the leading edge (containing more energy). This fact alone suggests that linearization might not be a good choice. Only for large integration periods of T_i= 8 ns the difference between linearization and operation directly on the correlator outputs is negligible for NLOS scenarios. In terms of robustness linearization has no influence - the difference between these two options is only noticable when the algorithm reaches approaches or hits the error floor. Numeric results for the noise-free case (i.e. the error floor) can be seen in Tab. 1.

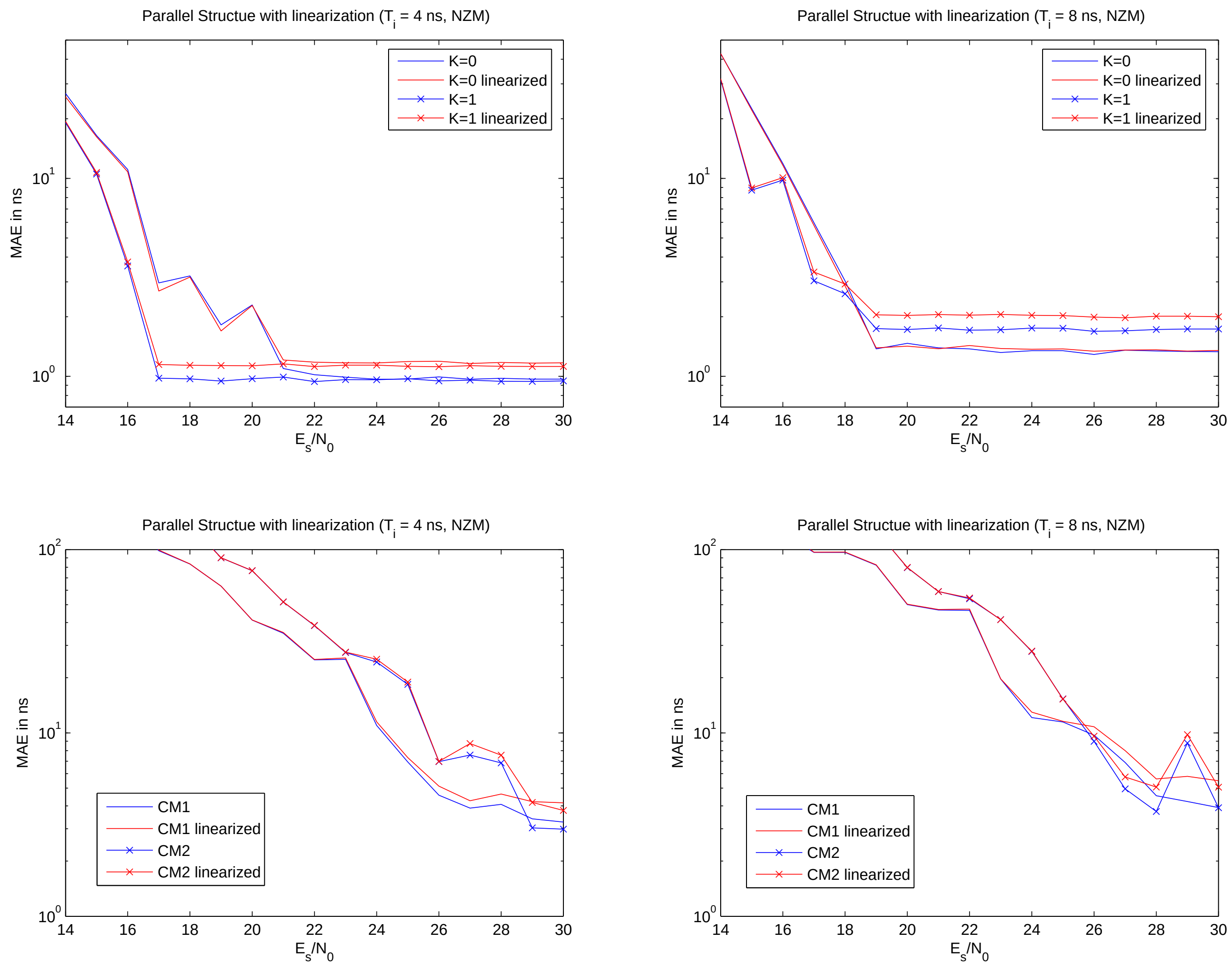

Figure 5: Ranging Accuracy for linear interpolation

1.3 Equalization

I already shows that the parallel structure can be related to the direct structure with MA filtering, so the idea of equalization comes at hand. Unfortunately, the zero-forcing (ZF) equalizer for a two-tap MA filter is instable, because its frequency response has a pole on the unit circle:

$$H_{MA}(z) = 1 + z^{-1} \tag{23}$$

$$H_{eq}(z) = \frac{1}{1+z^{-1}} = \frac{z}{z+1} \tag{24}$$

. Furthermore, the MMSE equalizer assumes that noise is introduced between the system and the equalizer, which is not the case in this scenario. This is problematic in the sense that

$$\frac{\sigma_n^2}{\sigma_x^2}\mathbf{I} + \mathbf{H}\mathbf{H}^H \tag{25}$$

is only guaranteed invertible if the noise variance σ_n^2 is positive (σ_x^2 is the energy of the signal and $\mathbf{H}$ is the convolution matrix of the impulse response to be equalized).

For verification of my ideas I persued two approaches: First, I tried to add a little artificial noise for the equalizer design only, and secondly I pulled the pole of the ZF equalizer a little into the unit circle. For the first approach I found that, naturally, the longer the MMSE equalizer is and the smaller the noise variance for design purposes is, the more the MMSE equalizer resembles the ZF equalizer – also in terms of oscillations. Since furthermore the MMSE equalizer requires two parameters to be optimized (filter order and design noise variance), I opted for the sub-optimum ZF equalizer with a pole pulled inside the unit circle. I thus obtained the following freuency response:

$$H_{eq}(z) = \frac{z}{z+0.9} \tag{26}$$

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I simulated the performance of the equalized parallel structure for the same set of parameters as the linear interpolation and obtained interesting results. As seen in Fig. 6, equalization yields better results for LOS channels and for high SNR regions for both long and short integration periods. On the other hand, robustness is decreased for medium SNR regions, since the equalized structure is more susceptible to noise. If, on the other hand, the noise is low, superior results can be achieved (as it can be seen in Tab. 1). The influence of LOS and NLOS on equalization is not yet fully clear to me, but it seems that equalization produces better results when the threshold is high, because the event that an oscillation produced by the equalizer qualifies as a TOA estimate is less likely.

The simulation for the IEEE CM1 and CM2 is not fully satisfying as well because obviously even higher SNR values would be required to approach the error floor. Please, consider these results preliminary.

	Parallel		Linearized		Equalized	
	SB (adapt.)	SB (norm.)	SB (adapt.)	SB (norm.)	SB (adapt.)	SB (norm.)
T_i= 2 ns						
ZM, k=0	0.866	0.88	1.143	0.88	0.703	0.78
NZM, k=0	0.871	0.88	1.152	0.88	0.705	0.79
ZM, k=1	0.902	0.94	1.089	0.94	0.537	0.54
NZM, k=1	0.908	0.95	1.096	0.95	0.539	0.54
ZM, CM1	2.444	2.48	3.725	2.48	2.149	2.13
NZM, CM1	2.454	2.49	3.737	2.49	2.149	2.13
ZM, CM2	2.709	2.72	3.646	2.72	2.241	2.26
NZM, CM2	2.721	2.73	3.673	2.73	2.244	2.27
T_i= 4 ns						
ZM, k=0	1.459	1.47	1.376	1.47	1.068	1.21
NZM, k=0	1.459	1.47	1.381	1.47	1.076	1.22
ZM, k=1	1.691	1.72	1.975	1.72	1.011	1.01
NZM, k=1	1.704	1.73	1.989	1.73	1.015	1.01
ZM, CM1	3.296	3.34	5.611	3.34	2.339	2.59
NZM, CM1	3.324	3.36	5.631	3.36	2.347	2.60
ZM, CM2	3.299	3.33	5.225	3.33	2.817	3.12
NZM, CM2	3.346	3.34	5.253	3.34	2.844	3.13

Table 1: Ranging Results for T_i=2ns and various algorithms

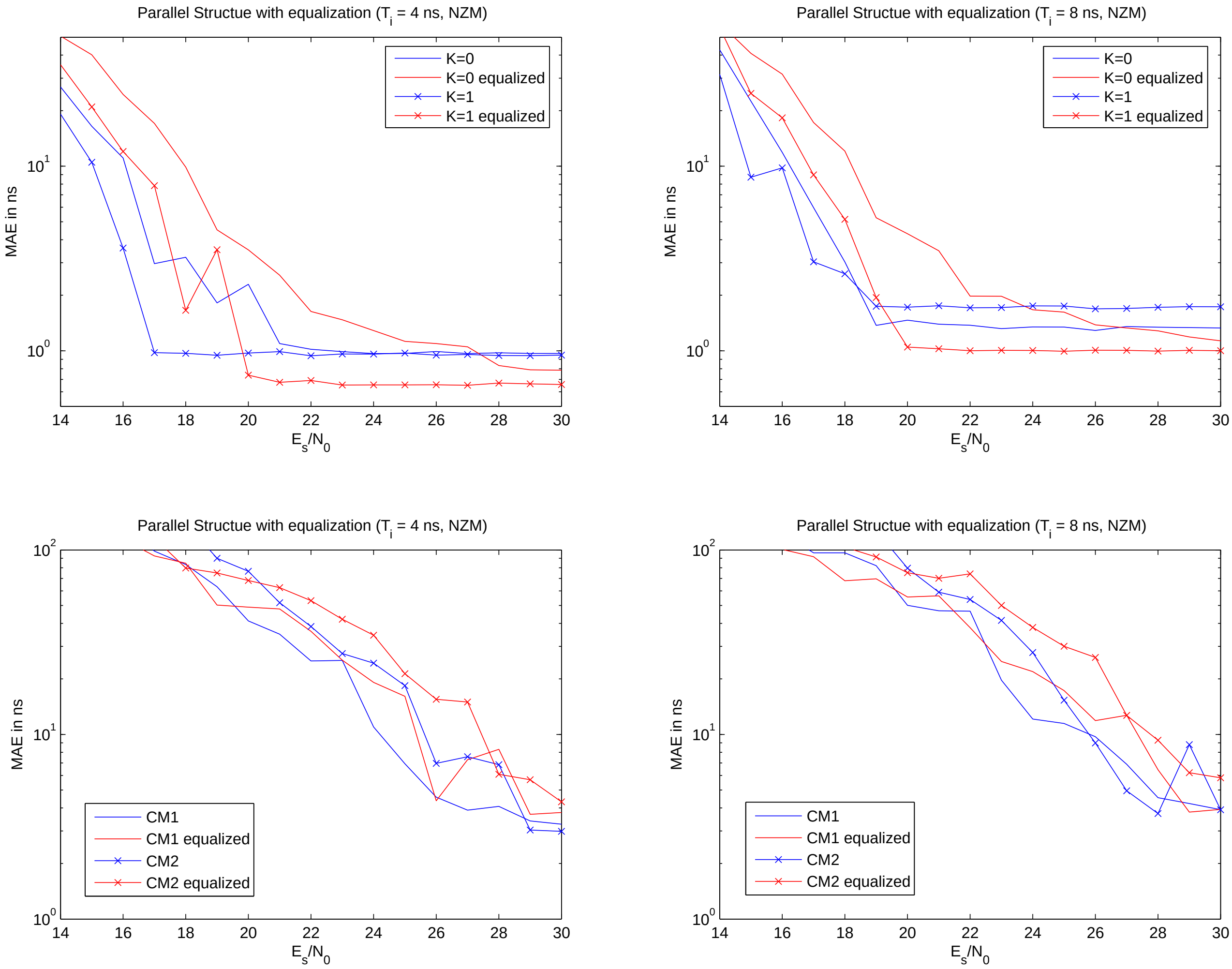

Figure 6: Ranging Accuracy for equalization